Renormalizable elektroweak model without fundamental scalar mesons.

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Abstract

A renormalizable model of electroweak interaction which coincides with Weinberg-Salam model in the gauge boson - fermion sector but does not require the existence of fundamental scalar fields is proposed.

1 Introduction

The Weinberg-Salam model [1], [2] gives an excellent description of electroweak processes in the framework of renormalizable field theory. The essential ingredient of this model is the scalar field providing spontaneous breaking of $SU(2) \times U(1)$ symmetry via Higgs mechanism [3], [4], which allows to generate a mass term for the vector boson without breaking the gauge invariance.

Predictions of the Weinberg-Salam model for the fermion-vector meson sector are in a very good agreement with experiment. However all attempts to find experimentally the scalar Higgs meson up to now failed. Of course it is possible that such meson will be found on LHC or other big machines. Nevertheless it seems worth to look for an alternative model which preserves good predictions of the Weinberg-Salam model in the fermion-gauge meson sector but does not require existence of an elusive spin zero boson. Such a model is proposed in this paper.

2 Higher derivative reformulation of the Higgs-Kibble model.

We start with reformulation of the usual $SU(2)$ Higgs-Kibble model which is suitable for generalization described further. Such a formulation was given in our paper [5].

It is based on the higher derivative action

$$A = \int dx \{ L_{YM} + (D_\mu \varphi)^+ (D_\mu \varphi) +$$
$$\frac{g}{2m} \partial_\mu X \partial_\mu (\varphi^+ \varphi) + \frac{a^{-2}}{2} \Box X \Box X + \partial_\mu \bar{c} \partial_\mu c \} \tag{1}$$

Here L_{YM} is the $SU(2)$ Yang-Mills Lagrangian, φ is the complex doublet with the components

$$\varphi_1 = \frac{iB_1 + B_2}{\sqrt{2}}; \quad \varphi_2 = \frac{\sqrt{2}m}{g} + \frac{1}{\sqrt{2}}(\sigma - iB_3) \tag{2}$$

m is the mass which the Yang-Mills quanta acquire via Higgs mechanism, D_μ is the usual covariant derivative and $\bar{c}, c$ are anticommuting ghost fields.

Under the gauge transformations the Yang-Mills field A_μ and the scalar fields φ transform in a standard way and the fields X and $\bar{c}, c$ are singlets of the gauge group. Due to the presence of the constant term in the component φ_2, the quadratic part of this action is not diagonal with respect to the fields X and σ. To diagonalize it we make a shift $\sigma \to \sigma - X$. The action (1) acquires a form

$$A = \int dx \{ \tilde{L}(A_\mu, B, \tilde{\sigma}) + \frac{1}{2} \partial_\mu \sigma \partial_\mu \sigma - \frac{1}{2} \partial_\mu X \partial_\mu X +$$
$$\frac{a^{-2}}{2} \Box X \Box X + \partial_\mu \bar{c} \partial_\mu c + \frac{g}{4m} \partial_\mu X \partial_\mu (B^2 + \tilde{\sigma}^2) \} \tag{3}$$

Here $\tilde{L} + \frac{1}{2} \partial_\mu \sigma \partial_\mu \sigma$ is the usual Lagrangian for the Yang-Mills field interacting with B and σ, in which σ in the interaction is replaced by $\tilde{\sigma} = \sigma - X$.

It was shown in our paper [5] that quantization of this model with the help of Ostrogradsky canonical formalism in the gauge $B = 0$ leads to the following spectrum: three massive spin one particles associated with the fields A_μ, and one spin zero massless field σ. The field X generates two states: one massive spin zero particle with the mass a and one massless spin zero state with negative norm. The corresponding propagators look as follows

$$D_{\mu\nu}^{ab} = \frac{g^{\mu\nu} - k^\mu k^\nu m^{-2}}{k^2 - m^2} \delta^{ab} \tag{4}$$

$$D_\sigma = \frac{1}{k^2} \tag{5}$$

$$D_X = \frac{a^2}{k^2(k^2 - a^2)} \tag{6}$$

Due to invariance of the action (3) with respect to the supersymmetry transformations

$$\delta X = c\varepsilon; \quad \delta \sigma = c\varepsilon$$
$$\delta \bar{c} = X\varepsilon - \sigma\varepsilon - a^{-2} \Box X - \frac{g}{4m}(B^2 + \tilde{\sigma}^2) \tag{7}$$

there exists a conserved operator Q, which separates the invariant subspace

$$Q|\Phi >= 0 \tag{8}$$

with the nonnegative norm [5]. The explicit solution of eq. (8) has a form

$$|\Phi >= [1 + \sum_n c_n \prod_{1\leq j\leq n} (\sigma^+(k_j) - a^+(k_j))]|\Phi >_{A,b} \tag{9}$$

where σ^+ and a^+ are the creation operators of the positive and negative norm massless states respectively, and $|\Phi >_{A,b}$ describes the states with the massive vector field and the scalar particle with the mass a.

The spectrum of observables in this model is identical to the Higgs model, and it is easy to show that the transition amplitudes also coincide. Explicitly renormalizable perturbation theory may be constructed by passing from the gauge $B = 0$ to some renormalizable gauge like the Lorentz gauge $\partial_\mu A_\mu = 0$.

It is worth to note that in distinction of the standard Higgs model, where in the gauge $B = 0$ only physical states are present and physical unitarity is manifest, in our formulation even in the $B = 0$ gauge unphysical states are present and the unitarity is achieved by imposing on the physical vectors the condition (8).

The unitarity of the model is easily understood if one looks at the structure of the X-field propagators (6):

$$\tilde{D}_X = \frac{a^2}{k^2(k^2 - a^2)} = \frac{1}{k^2 - a^2} - \frac{1}{k^2} \tag{10}$$

The field X enters the interaction in the action (3) either in combination $\tilde{\sigma} = \sigma - X$ or in the form $\Box X$. In the first case the contribution of the negative norm component of X exactly compensates the contribution of the massless excitation σ, and in the second case due to the presence of D'Alambert operator the negative norm excitations are not created in the asymptotic states.

Although the physical content of the model described above is equivalent to the Higgs model, it is more convenient for generalizations including additional scalar particles. Contrary to the Higgs model where the physical scalar particles belong to multiplets transforming nontrivially under the gauge transformations, in our formulation the physical scalar particle is the gauge singlet. It allows to introduce additional terms depending on X-fields without breaking the gauge invariance of the model.

3 A new family of renormalizable vector meson models with spontaneously broken symmetry.

Let us consider the model described by the following action

$$A = \int dx[L_{YM} + (D_\mu\varphi)^+(D_\mu\varphi) + \frac{g}{2m}\partial_\mu(\sum_{i=1-N/2}^{N/2} X_i)\partial_\mu(\varphi^+\varphi) +$$
$$\frac{1}{2}(\sum_{i=1-N/2}^{N/2} \partial_\mu X_i)^2 - \frac{N}{2}\sum_{i=1-N/2}^{N/2} \partial_\mu X_i \partial_\mu X_i + \frac{N}{2}\sum_{i=1-N/2}^{N/2} a_i^{-2}\Box X_i \Box X_i] \tag{11}$$

We did not write here the ghost fields $\tilde{c}, c$ which are free and do not influence the results. The complex doublet field φ is parameterized as in eq.(2).

Having in mind that the fields X_i are singlets of the gauge group we see that the action (11) is gauge invariant.

To diagonalize the quadratic part of this action we again make a shift

$$\sigma \rightarrow \sigma - \sum_{i=1-N/2}^{N/2} X_i \tag{12}$$

Then the action (11) acquires a form

$$A = \int dx\{\tilde{L}(A_\mu, B, \tilde{\sigma}) + \frac{1}{2}\partial_\mu\sigma\partial_\mu\sigma - \frac{N}{2}\sum_i \partial_\mu X_i \partial_\mu X_i + \\ \frac{N}{2}\sum_i a_i^{-2}\Box X_i \Box X_i + \frac{g}{4m}\partial_\mu(\sum_i X_i)\partial_\mu(B^2 + \tilde{\sigma}^2) \tag{13}$$

Here as in the eq.(3) $\tilde{L} + \frac{1}{2}\partial_\mu\sigma\partial_\mu\sigma$ is the usual Lagrangian for the Yang-Mills field interacting with B and σ with the substitution $\sigma \rightarrow \tilde{\sigma}$.

$$\tilde{\sigma} = \sigma - \sum_{i=1-N/2}^{N/2} X_i \tag{14}$$

Imposing the gauge condition $B^a = 0$ one easily finds the propagators of the fields A_μ, σ, X_i. The propagators of A_μ and σ are given by the eqs.(4, 5), and the propagators of X_i are equal to

$$\tilde{D}_{ij}^X(k) = \delta_{ij}\frac{a_i^2}{Nk^2(k^2 - a_i^2)} \tag{15}$$

Hence the propagators of $\tilde{\sigma}$ are

$$\tilde{D}_{\tilde{\sigma}}(k) = \frac{1}{k^2} + \sum_{i=1-N/2}^{N/2} \frac{a_i^2}{Nk^2(k^2 - a_i^2)} = \sum_{i=1-N/2}^{N} \frac{1}{N(k^2 - a_i^2)} \tag{16}$$

One sees that the propagators of $\tilde{\sigma}$ have no zero mass poles. The field σ exactly compensates the contribution of zero mass components of X and only positive norm components with the masses a_i are propagating. As in the previous model the interaction terms include the fields X_i only in the combinations $\sigma - \sum_i X_i$ or $\Box X_i$. Neither of these combinations generates negative norm states and the theory is unitary in the physical subspace including only massive vector fields and the massive components of the fields X_i.

The interaction of the scalar particles is suppressed by the factor $(N)^{-1}$ and for large N the probability of creation of a zero spin particle with the fixed mass a_i is very small. At the same time in the intermediate states all massive components of X_i fields contribute and for $|k| \gg a_i$, the propagator $\tilde{D}_{\tilde{\sigma}}(k)$ coincides with the usual

Higgs meson propagator. However for $k \sim a_i$ the interaction of scalar particles may differ considerably from the Higgs meson interaction.

Obviously one can choose instead of the gauge $B^a = 0$ some other gauge condition, for example the Lorentz gauge $\partial_\mu A_\mu = 0$. In this gauge the vector field propagator decreases at $k \to \infty$ as k^{-2} and there are additional B-field propagators, which also decrease as k^{-2}. In this gauge the model is manifestly renormalizable.

As we discussed before for large values of N creation of one particle spin zero states is suppressed, but contribution of all scalar particles produces a collective effect which replaces the Higgs meson exchange in the Weinberg-Salam model. This collective effect may be attributed to the manifestation of a special extra dimension.

Indeed, the model described above may be considered as having a discrete extra dimension. The Yang-Mills fields and the fields φ are living on the four-dimensional "brane", whereas the fields X_i and their masses depend on the fifth coordinate.

Continuous version of such a model may be described by the action

$$\begin{aligned} A = \int dx[L_{YM} + (D_\mu\varphi)^+(D_\mu\varphi) + \\ \frac{g}{2m}\partial_\mu(\varphi^+\varphi)\int_{-\pi/\kappa}^{\pi/\kappa}\partial_\mu X(x,\lambda)d\lambda + \frac{1}{2}\int_{-\pi/\kappa}^{\pi/\kappa}\partial_\mu X(x,\lambda)d\lambda\int_{-\pi/\kappa}^{\pi/\kappa}\partial_\mu X(x,\lambda)d\lambda + \\ \frac{\pi}{\kappa}\int_{-\pi/\kappa}^{\pi/\kappa}\Box X(x,\lambda)\Box X(x,\lambda)a^{-2}(\lambda)d\lambda - \frac{\pi}{\kappa}\int_{-\pi/\kappa}^{\pi/\kappa}\partial_\mu X(x,\lambda)\partial_\mu X(x,\lambda)d\lambda] \end{aligned} \tag{17}$$

where φ is again parameterized as in eq.(2).

Diagonalizing the quadratic part of the action (17) by the shift

$$\sigma(x) \to \sigma(x) - \int_{-\pi/\kappa}^{\pi/\kappa} X(x,\lambda)d\lambda \tag{18}$$

we get

$$\begin{aligned} A = \int dx[L(A_\mu, B, \tilde{\sigma}) + \frac{1}{2}\partial_\mu\sigma\partial_\mu\sigma + \\ \frac{g}{4m}\partial_\mu(B^2+\tilde{\sigma}^2)\int_{-\pi/\kappa}^{\pi/\kappa}\partial_\mu X d\lambda - \frac{\pi}{\kappa}\int_{-\pi/\kappa}^{\pi/\kappa}\partial_\mu X\partial_\mu X d\lambda + \\ \frac{\pi}{\kappa}\int_{-\pi/\kappa}^{\pi/\kappa}\Box X\Box X a^{-2}(\lambda)d\lambda] \end{aligned} \tag{19}$$

Here

$$\tilde{\sigma}(x) = \sigma(x) - \int_{-\pi/\kappa}^{\pi/\kappa} X(x,\lambda)d\lambda \tag{20}$$

The X-field propagator defined by the action (19) is

$$\tilde{D}(k,\lambda,\mu) = \delta(\lambda-\mu)\frac{\kappa}{2\pi}\frac{a^2(\lambda)}{k^2(k^2-a^2(\lambda))} \tag{21}$$

Discretized version corresponds to $\frac{2\pi}{\kappa} = Nb$, where b is the lattice spacing.

The propagator of the field $\tilde{\sigma}$ looks as follows

$$\tilde{D}_{\tilde{\sigma}}(k)=\frac{1}{k^2}+\int_{-\pi/\kappa}^{\pi/\kappa}d\lambda d\mu\frac{\kappa}{2\pi}[-\frac{1}{k^2}+\frac{1}{k^2-a^2(\lambda)}]\delta(\lambda-\mu)=$$
$$\frac{\kappa}{2\pi}\int_{-\pi/\kappa}^{\pi/\kappa}d\lambda\frac{1}{k^2-a^2(\lambda)} \tag{22}$$

It has no pole at $k^2=0$ and for $k^2\gg a^2$ $\tilde{D}_{\tilde{\sigma}}$ coincides with the Higgs meson propagator.

As an example one can take

$$a^2(\lambda)=\lambda+\frac{\pi}{\kappa}+M_0^2,\quad \lambda<0;\quad a^2(\lambda)=-\lambda+\frac{\pi}{\kappa}+M_0^2,\quad \lambda>0 \tag{23}$$

Then

$$\tilde{D}_{\tilde{\sigma}}(k)=-\frac{\kappa}{\pi}\ln(1-\frac{\pi}{\kappa(k^2-M_0^2)}) \tag{24}$$

The propagators of the fields $\tilde{\sigma}$ and $\Box X$ have no one particle pole, but generate a branch point corresponding to the collective excitations. One sees that for $k^2\gg\frac{\pi}{\kappa}+M_0^2$ the function $\tilde{D}_{\tilde{\sigma}}$ coincides with the usual Higgs meson propagator, $\tilde{D}\sim k^{-2}$, but for small k^2 the behavior of $\tilde{D}_{\tilde{\sigma}}$ is quite different.

4 Electroweak model.

The mechanism described in the previous section may be applied directly to the description of electroweak interactions. The fermion-gauge meson sector coincides with the Salam-Weinberg model, but the Higgs sector is described by the Lagrangian

$$L=|\partial_\mu\varphi+ig\frac{\tau^a}{2}A_\mu^a\varphi-\frac{ig_1}{2}B_\mu\varphi|^2-G(\bar{L}\varphi R+\bar{R}\varphi L+\ldots)$$
$$+\frac{g}{2m}\sum_i\partial_\mu X_i\partial_\mu(\varphi^+\varphi)+\frac{1}{2}\sum_i\partial_\mu X_i\sum_i\partial_\mu X_i-$$
$$\frac{N}{2}\sum_i\partial_\mu X_i\partial_\mu X_i+\frac{N}{2}\sum_i a_i^{-2}\Box X_i\Box X_i \tag{25}$$

where summation over i goes from $1-N/2$ to $N/2$. Letters L,R denote lepton doublets and singlets and $\ldots$ stand for the corresponding terms with quark fields. The field φ is parameterized as before

$$\varphi_1=\frac{(B_1+iB_2)}{\sqrt{2}};\quad \varphi_2=\frac{\sqrt{2}m}{g}+\frac{\sigma-iB_3}{\sqrt{2}} \tag{26}$$

and A_μ^a, B_μ are the gauge fields corresponding to $SU(2)$ and $U(1)$ subgroups.

The Lagrangian (25) is gauge invariant if the fields $A_\mu, B_\mu, \varphi, L, R$ transform in a usual way and the X_i fields are the gauge group singlets.

The constant component of φ_2 generates mass terms for gauge fields and fermions and produces mixing between the fields X_i and σ. Making the shift (14) we obtain

the model whose gauge boson - fermion sector is identical to the Weinberg-Salam model, and the scalar mesons are described by the action (11) with obvious modifications due to the presence of the $U(1)$ gauge field B_μ. The interaction with fermions enters via the terms

$$G[\bar{L}(\varphi - \sum_i X_i)R + h.c. + \ldots] \tag{27}$$

In the limit $N \to \infty$ the model may be described by the action of the type (17) with continuous extra dimension. In this case one particle poles describing spin zero particles completely disappear from the spectrum and instead there is a branch point corresponding to some collective excitations. For a finite N our model describes the system with several neutral spin zero particles.

5 Conclusion.

In this paper we described the renormalizable gauge invariant model of massive Yang-Mills field, which does not require the existence of fundamental scalar particles. A discretized version describes a gauge invariant model of massive vector field with spontaneously broken symmetry and several neutral scalar mesons. This mechanism may be applied to the electroweak model of the Salam-Weinberg type to modify the predictions concerning spin zero particles. We considered here in details only the $SU(2)$ group as being the most important for electroweak models, but all the results are trivially generalized to other gauge groups.

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